98

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Exam II

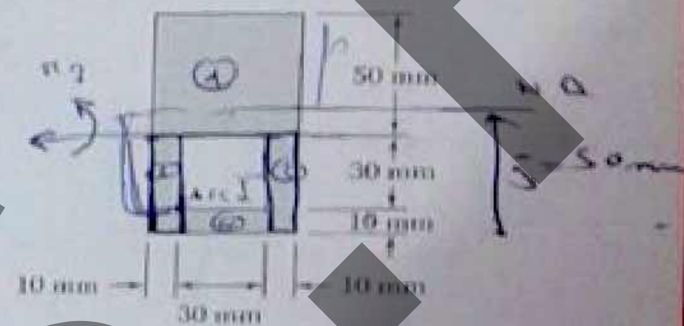
Friday May 16, 2014
Duration: 1h 30min
All work **must** be shown to receive full credit.

Problem 1. (20 pts.) A beam of the cross section shown is extruded from an aluminum alloy. If the moment applied about the z axis (horizontal axis directed to the left) is $M_z = 10$ kN.m, calculate:

a - The maximum tensile stress. (12 pts.)

b - The maximum compressive stress. (4 pts.)

c - The stress at point A. (4 pts.)



a) $\sigma_{max} = \frac{M c_{max}}{I}$

$\bar{y} = \frac{\bar{y}_{rectangle} A_{rectangle} - \bar{y}_{hole} A_{hole}}{A_{rectangle} - A_{hole}}$

$\bar{y} = \frac{45 \times 90 \times 50 - 25 \times 30 \times 30}{90 \times 50 - 30 \times 30}$

$\bar{y} = \frac{202500 - 22500}{4500 - 900} = 50$ mm

4

$I = I_1 + 2 I_2 + I_4$

$I_1 = \frac{1}{12} b h^3 + A d^2 = \frac{1}{12} \times 50 \times 10^{-3} \times (50 \times 10^{-3})^3 + 50 \times 50 \times 10^{-6} \times (15 \times 10^{-3})^2$

$= 5.208 \times 10^{-7} + 5.625 \times 10^{-7} = 1.0833 \times 10^{-6}$ m^4

$I_2 = 2\left[\frac{1}{12} b h^3 + A d^2\right] = 2\left[\frac{1}{12} \times 10 \times 10^{-3} \times (40 \times 10^{-3})^3 + 40 \times 10 \times 10^{-6} \times (30 \times 10^{-3})^2\right]$

$= 2\left[5.33 \times 10^{-8} + 3.6 \times 10^{-7}\right]$

$= 8.266 \times 10^{-7}$ m^4

4

20

$$I_4 = \frac{1}{12} bh^3 + Ad^2 = \frac{1}{12}\, 30\times10^{-3}\times(10\times10^{-3})^3 + 30\times10\times10^{-6}\times(45 \ldots$$

$$= 2,5\times10^{-9} + 6\,075\times10^{-7}$$

$$= 6,1\times10^{-7}\,m^4$$

$$I = I_1 + I_2 + I_4$$

$$\boxed{I = 2,52\times10^{-6}\,m^4}$$

a) tension occurs at bottom :

$$\sigma = -\frac{Mc_{ten}}{I} = -\frac{10\times10^{3}\times(-50\times10^{-3})}{2,52\times10^{-6}} = 198,412\times10^{6}\,Pa$$

$$= 198,412\ MPa$$

4

$$\sigma_{tension\,max} = |198,412|\ MPa$$

b) compression at top :

$$\sigma_{top} = -\frac{Mc_{top}}{I} = -\frac{10\times10^{3}\times40\times10^{-3}}{2,52\times10^{-6}} = -158,73\ MPa$$

$$\sigma_{top\,max} = |-158,73|\ MPa$$ 4

$$\sigma_{comp\,max} = 158,73\ MPa$$

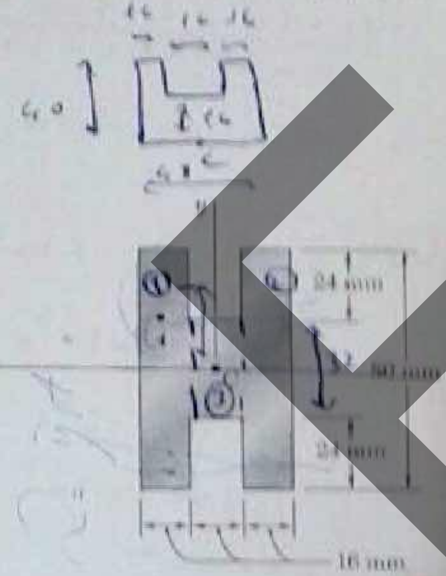
c) $$\sigma_a = -\frac{Mc_a}{I} = -\frac{10\times10^{3}\times(-40\times10^{-3})}{2,52\times10^{-6}}$$ 4

$$= 158,73\ MPa$$

Problem 2. (20 pts.) The cross section shown is subjected to a shear force of 10 kN directed along the y axis. Calculate:

a - The maximum shear stress at A. (12 pts.)

b - The shear stress at C. (8 pts.)



due to symmetry

$\bar{y} = 40$ mm

$I = I_1 + I_2 + I_3$

$I_1 = I_2 = \frac{1}{12} \times 16\times10^{-3} (80\times10^{-3})^3$

$= 6.8267\times10^{-7}\ m^4$

$I_3 = \frac{1}{12} b h^3 = \frac{1}{12} \times 16\times10^{-3} \times (32\times10^{-3})^3$

$= 4.37 \times10^{-8}\ m^4$

$I = 2 I_1 + I_3 = 2\times(6.8267\times10^{-7}) + 4.37\times10^{-8}$

$I = 1.40904 \times10^{-6}\ m^4$

a) since A is located above N.A., find Area above point A × 2

$Q_A = 2 \times A \times \bar{y}$

$= 2 \times 16\times24\times10^{-6} \times 28\times10^{-3}$

$= \cancel{1.2288\times10^{-5}}\ m^3$ $\quad Q_A = 2.1504\times10^{-5}\ m^3$

$t_A = 16\times10^{-3} \times 2 = 0.032\ m$

20

~~$\tau_A = \frac{V Q_A}{I t_A} = \frac{40\times10^{3} \times 1{,}7298\times10^{-5}}{0{,}032 \times 1{,}403046\times10^{-6}}$~~

~~$\tau_A = 2{,}7253 \text{ MPa}$~~

$$= \frac{10\times10^{3} \times 2{,}1504\times10^{-5}}{0{,}032 \times 1{,}40304\times10^{-6}} = 4{,}77 \text{ MPa}$$

b) find Q for Area above C for max value

$$Q_C = A_1 \bar{y}_1 + 2 A_2 \bar{y}_2$$

$$= 48\times16\times10^{-6} \times 8\times10^{-3} + Q_A$$

$$= 6{,}144\times10^{-6} + 2{,}1504\times10^{-5}$$

$$= 2{,}7648\times10^{-5} \text{ m}^3$$

$t = 48\times10^{-3}$ m

$$\tau_C = \frac{V Q_C}{I t_C} = \frac{10\times10^{3} \times 2{,}7648\times10^{-5}}{48\times10^{-3} \times 1{,}40304\times10^{-6}}$$

$$= 4{,}088 \text{ MPa}$$

Problem 3. (20 pts.) The beam shown below has a rectangular cross-sectional area of 0.2 × 0.4 m^2.

a - Draw the shear and bending moment diagrams. (14 pts.)

b - Calculate the maximum shear stress in the beam. (6 pts.)

a)

$\circlearrowright \Sigma M_C = 0$;

$-12 \times 5.2 + R_B \times 4 - 16 \times 4 \times 2 = 0$

$\boxed{R_B = 47.6 \text{ kN}}$

$\uparrow \Sigma F = 0$;

$-12 + 47.6 - 16 \times 4 + R_C = 0$

$\boxed{R_C = 28.4 \text{ kN}}$

$V_A = -12$ kN

$V_B^- = V_A - \int w\,dx$
$= V_A$

$V_B^+ = V_A + R_B$
$= 35.6$ kN

$V_C = V_B^+ - \int w\,dx$
$= 35.6 - 16 \times 4$
$= -28.4$ kN

$M_A = 0$

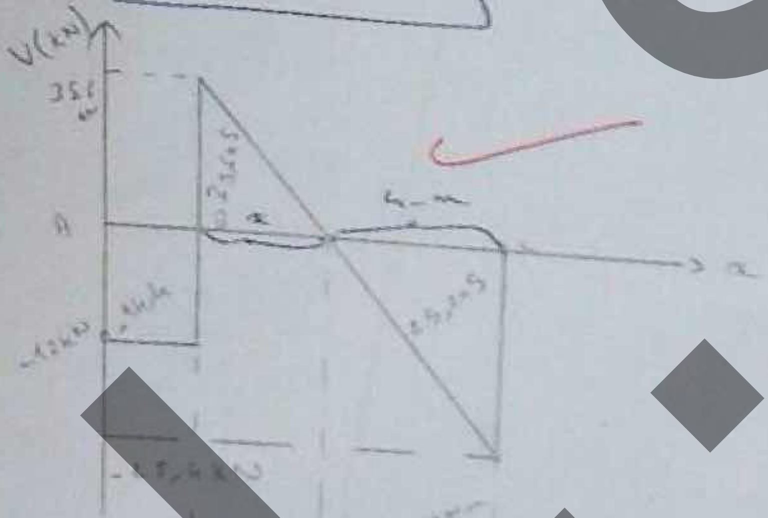
$\frac{x}{35.6} = \frac{4-x}{28.4}$

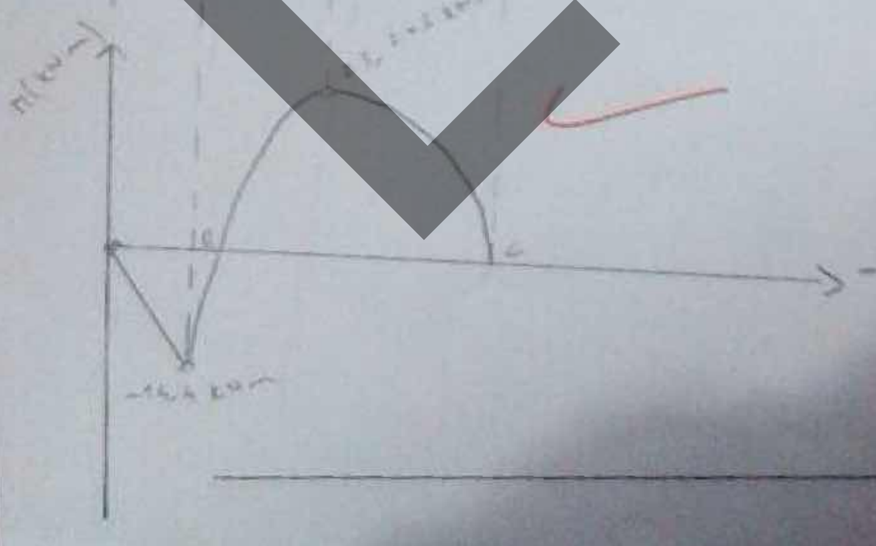
$28.4x = 35.6(4-x)$

$28.4x = 142.4 - 35.6x$

$64x = 142.4$

$\boxed{x = 2.225 \text{ m}}$





$M_B = -14.4$ kN·m

$M_{max} = -14.4 + 39.605$
$= 25.205$ kN·m

$M_C = 0$

20

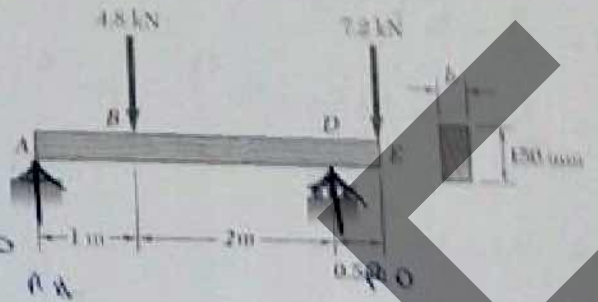
b) $V_{max} = |75,6\ kN|$ at B

τ_{max} for rectangular cross section

$$\tau_{max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2} \times \frac{75,6\times10^3}{0,2\times0,4}$$

$$= 667500\ Pa$$

Problem 4. (20 pts.) For the beam and loading shown, determine the minimum required width b, knowing that for the grade of timber used, $\sigma_{all} = 14$ MPa and $\tau_{all} = 900$ kPa.



$\circlearrowleft \Sigma M_D = 0$

$R_A \times 3 - 4.8 \times 2 + 7.2 \times 0.5 = 0$

$\boxed{R_A = 2\ kN}$

$\uparrow \Sigma F = 0$

$R_A - 4.8 - 7.2 + R_D = 0$

$\boxed{R_D = 10\ kN}$

$M_A = M_E = 0$

$V_A = 2\ kN$

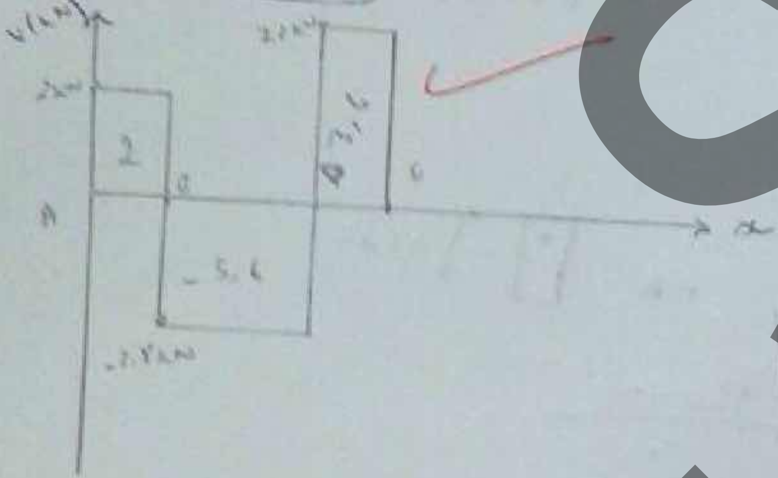
$V_B^- = 2\ kN$

$V_B^+ = 2 - 4.8 = -2.8\ kN$

$V_D^- = -2.8\ kN$

$V_D^+ = -2.8 + 10 = 7.2\ kN$

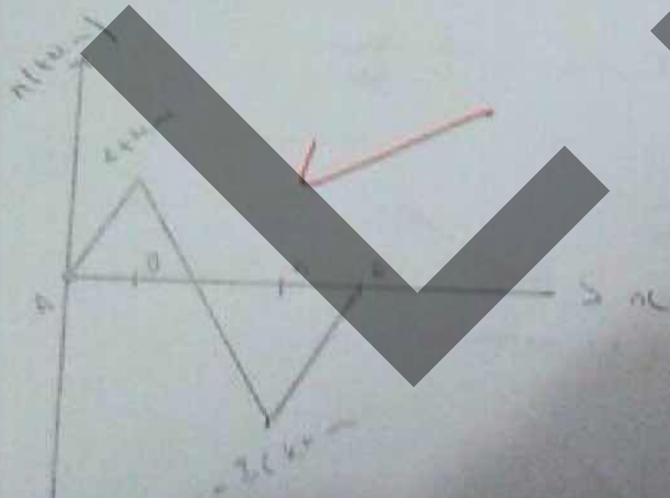
$V_E = 7.2 - 7.2 = 0$



$M_B = 2\ kN \cdot m$

$M_D = 2 - 5.6 = -3.6\ kN \cdot m$

$M_E = -3.6 + 3.6 = 0$



$V_{max} = |7.2\ kN|$

$M_{max} = |3.6\ kN \cdot m|$ at D

↳ negative ⇒ tension at top
compression at bottom

20

$$\sigma = \frac{Mc}{I}$$

$$I = \frac{1}{12} b h^3$$
$$= \frac{1}{12} b \times (150 \times 10^{-3})^3 \ m^4$$

$$c = 75 \times 10^{-3} \ m$$

$$14 \times 10^6 = \frac{3{,}6 \times 10^3 \times 75 \times 10^{-3}}{\frac{1}{12} b \ (150 \times 10^{-3})^3}$$

$$b = \frac{12 \times 3{,}6 \times 10^3 \times 75 \times 10^{-3}}{14 \times 10^6 \times (150 \times 10^{-3})^3} = \frac{3240}{47250} = 68{,}57 \ mm$$

$$\tau = \frac{VQ}{It}$$

$$Q_{max} = \frac{150 \times 10^{-3}}{2} \times b \times \frac{150 \times 10^{-3}}{4}$$

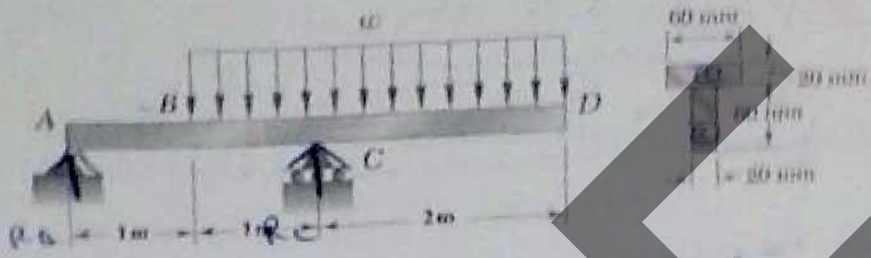
rectangular cross section $\Rightarrow \tau_{max} = \frac{3}{2} \frac{V}{A}$

$$300 \times 10^3 = \frac{3 \times 7{,}2 \times 10^3}{2 \times 150 \times 10^{-3} \times b}$$

$$b = \frac{21{,}6 \times 10^3}{300 \times 10^{-3} \times 300 \times 10^3} = 80 \ mm$$

$$\boxed{b = 80 \ mm}$$

Problem 5. (20 pts.) Determine the largest permissible distributed load w for the beam and loading shown, knowing that the allowable normal stress in tension is $\sigma_{T_{all}} = 160$ MPa and in compression $\sigma_{C_{all}} = -130$ MPa.



$$\bar{y} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A_1 + A_2} = \frac{70 \times 60 \times 20 + 30 \times 20 \times 60}{60 \times 20 + 60 \times 20}$$

$$= 50 \text{ mm}$$

$$I = I_1 + I_2$$

$$I_1 = \frac{1}{12} \times 60 \times 10^{-3} \times (20 \times 10^{-3})^3 + 60 \times 20 \times 10^{-6} (20 \times 10^{-3})^2$$

$$I_1 = 4 \times 10^{-8} + 4.8 \times 10^{-7} = 5.2 \times 10^{-7} \text{ m}^4$$

$$I_2 = \frac{1}{12} \times 20 \times 10^{-3} \times (60 \times 10^{-3})^3 + 60 \times 20 \times 10^{-6} (20 \times 10^{-3})^2$$

$$I_2 = 3.6 \times 10^{-7} + 4.8 \times 10^{-7} = 8.4 \times 10^{-7} \text{ m}^4$$

$$I = I_1 + I_2 = 5.2 \times 10^{-7} + 8.4 \times 10^{-7}$$

$$\boxed{I = 1.36 \times 10^{-6} \text{ m}^4}$$

18

$\curvearrowleft \Sigma M_C = 0$;

$R_A \times 2 - w \times 1 \times 0{,}5 + w \times 2 \times 1 = 0$

$2 R_A = -1{,}5\, w$

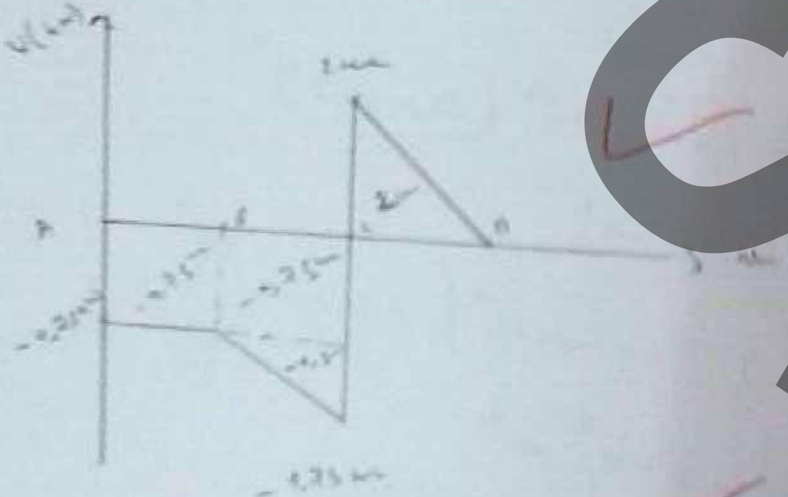
$\boxed{R_A = -0{,}75\, w}$ $(\downarrow)$

$\uparrow \Sigma F = 0$

$-R_A - w \times 3 + R_C = 0$

$-0{,}75\, w - 3w = -R_C$

$\boxed{R_C = 3{,}75\, w}$



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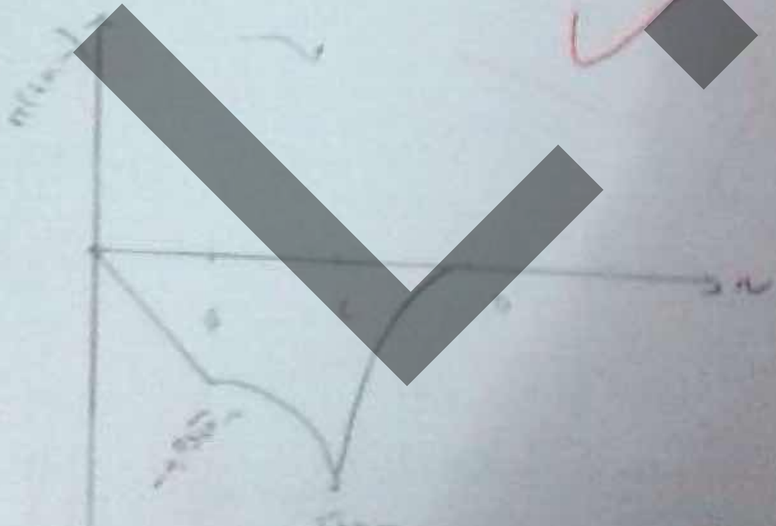

$M = |0.75w|$

$\sigma_T = \frac{M c_{top}}{I}$

$M = \frac{\sigma_T I}{c_{top}} = \frac{160\times10^6 \times 1.36\times10^{-6}}{20\times10^{-3}} = 0.75w$

$\boxed{w_1 = 14506.6\ N}$

$\sigma_c = \frac{M c_{bottom}}{I} =$ ~~0.75w~~

$M = \frac{\sigma_c I}{c} = 0.75w = \frac{-130\times10^6 \times 1.36\times10^{-6}}{-50\times10^{-3}}$

$\boxed{w_2 = 4714.667\ N}$

$M = |2w|$

$\sigma_T = \frac{M_T C}{I} = 2w$

$w_3 = \frac{\sigma_T I}{C_{top} \times 2} = \boxed{5440\ N}$

-1

answer

$w_4 = \frac{\sigma_c I}{c_{bottom} \times 2} = \boxed{1768\ N}$

-1

$\boxed{w_4 = 1768\ N}$